

Magnetic field at the center of a vortex: a new criterion for the classification of the superconductors

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Magnetic response of a superconductor depends on the thermodynamic stability of vortex in the material. Here we show that the vortex stability has a close relation with the ratio of the magnetic field at the vortex core center to the thermodynamic critical field. This finding provides a new criterion for the classification of the superconductors according to their magnetic responses.

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Type-1 and type-2 superconductors exhibit different magnetic responses to externally applied magnetic field. Whether there exists stable vortex is the distinction between two types of superconductors¹. Due to the formation of stable vortices, mix state appears in a type-2 superconductor under a certain range of external field. As a consequence of thermodynamic instability of vortex, only macroscopic Meissner state and fully normal state or the state of the coexistence of these two states (intermediate state) can exist in a type-1 material.

To date, the identifications of types of superconductors are based on the following three considerations: (1) to calculate the surface energy of a superconductor under the thermodynamic critical magnetic field²; (2) to compare the lower (higher) critical field at which the vortex entry into (exit from) the superconductor to the thermodynamic critical magnetic field^{3,4}; (3) to analyze the interaction between vortices⁵⁻¹¹. All these three considerations show that, the magnetic response of a superconductor is determined by a dimensionless parameter κ , which is defined as the ratio of the penetration depth to the coherence length. And the critical value of the Ginzburg-Landau (GL) parameter $\kappa_c = 1/\sqrt{2}$ represents a boundary between two types of superconductors.

In the present work, we revisit the problem of the thermodynamic stability of the vortex in a superconductor. We show that, there is a simple and rigorous relation between the magnetic field at the center of a vortex and the thermodynamic stability of vortex. Concretely, when the magnetic field at the center of vortex is larger than the thermodynamic critical field of the material, vortex is unstable and the superconductor is of type-1. On the contrary, when the magnetic field at the vortex core center is smaller than the thermodynamic field, vortex is stable and the superconductor is of type-2. The critical value, while the field at the core center equals to the thermodynamic critical field, represents a boundary between two cases. Since the magnetic response of a superconductor is completely determined by the the stability of vortex, our finding can serve as a new criterion for the classification of superconductors.

Vortex solution can always be constructed in the phenomenological Ginzburg-Landau model for superconductivity.

Here we use the GL free energy¹²

$$f = f_{n0} + \frac{\hbar^2}{2m^*} (\nabla |\Psi|)^2 + \frac{m^* c^2}{32\pi^2 e^{*2} |\Psi|^2} (\nabla \times \mathbf{B})^2 + V(|\Psi|^2) + \frac{\mathbf{B}^2}{8\pi}, \quad (1)$$

where f_{n0} is the free energy density of the body in the normal state in the absence of the external field, e^* and m^* are the effective mass and charge of the Cooper pair, $|\Psi|$ is the modulus of the superconducting order parameter, \mathbf{B} is the magnetic field, $V(|\Psi|^2) = \alpha |\Psi|^2 + \beta/2 |\Psi|^4$. The free energy (1) is equivalent to the usual GL model in which the gauge potential \mathbf{A} and the order parameter Ψ are the functions to describe the superconductivity, and $\mathbf{B} = \nabla \times \mathbf{A}$. We consider an isolate vortex in an infinite sample, and use instead of the variable r , the function $|\Psi|$ and \mathbf{B} the dimensionless quantities $\rho = r/\lambda$, $|\psi| = |\Psi|/\Psi_0$, $B = |\mathbf{B}|/H_c$, where $\lambda = (m^* c^2 \beta / 4\pi e^{*2} |\alpha|)^{1/2}$ is the penetration depth, $\Psi_0 = (-\alpha/\beta)^{1/2}$, $H_c = (4\pi\alpha^2/\beta)^{1/2}$ is the thermodynamic critical magnetic field. Radially symmetric vortex solution can be found by solving the following equations

$$-\frac{1}{\kappa^2} \frac{1}{\rho} \frac{d}{d\rho} \left(\rho \frac{d|\psi|}{d\rho} \right) + \frac{1}{2|\psi|^3} \left(\frac{dB}{d\rho} \right)^2 - |\psi| + |\psi|^3 = 0, \quad (2)$$

$$\frac{1}{\rho} \frac{d}{d\rho} \frac{\rho}{|\psi|^2} \frac{dB}{d\rho} = B,$$

where $\kappa = \lambda/\xi$ is the GL parameter, $\xi = \hbar/\sqrt{2m^*|\alpha|}$ is the coherence length. Far from the vortex core, the order parameter approaches the ground state value and the magnetic field vanishes to guarantee the Meissner state,

$$|\psi(\infty)| = 1, \quad B(\infty) = 0. \quad (3)$$

The fact that the vortex carries a nontrivial topological charge leads to the important conclusion that the flux carried by the vortex is quantized in unit of hc/e^* . Here we consider an isolated vortex line enclosing a single flux quantum, which is expected to have the lowest

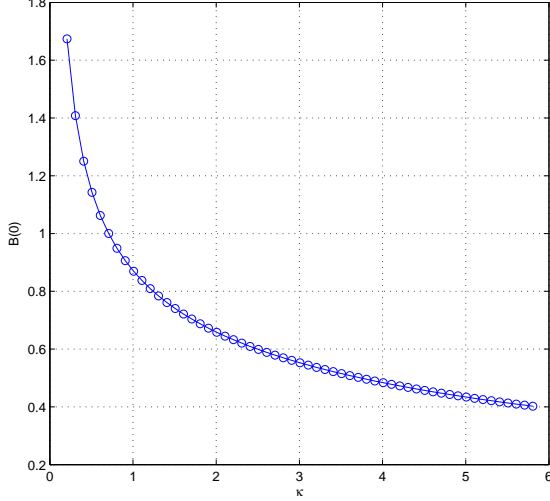


FIG. 1: (Color online) Magnetic field at the center of a vortex $B(0)$ as function of the Ginzburg-Landau parameter κ . $B(0) = 1$ corresponds to $\kappa = 1/\sqrt{2}$.

free energy. In this case, asymptotic behaviors of $|\psi|$ and B near the center of the vortex can be deduced from the quantization of flux $\int B \rho d\rho = \sqrt{2}/\kappa$ and Eqs. (2),

$$\begin{aligned} |\psi| &= a\rho + \dots, \\ B &= B(0) - \frac{a^2}{\sqrt{2}\kappa}\rho^2 + \dots, \end{aligned} \quad (4)$$

where a is the slope of $|\psi|$, $B(0)$ is the magnetic field at the center of the vortex. With boundary conditions of $|\psi|$ and B (3) and (4), Eqs. (2) can be solved. Then the vortex solution and a , $B(0)$ are determined¹³. Our results are more precise and comprehensive than those of previous work^{14,15}. In addition, precise solutions of GL vortex can also be obtained by using the iteration method developed by Brandt¹⁶.

We note that the difference of vortex solutions originates from the difference of the GL parameter κ . This means that $B(0)$, the magnetic field at the center of vortex, should be a function of the GL parameter κ ,

$$B(0) = B(0)(\kappa). \quad (5)$$

Further, considering the fact that the flux quantization is independent of the GL parameter κ , we then conclude that the magnetic field at the vortex center, $B(0)$, must be a monotonic function of the GL parameter κ .

In figure 1, magnetic field at the center of a vortex as a monotonic decreasing function of the GL parameter κ is shown. The critical value $\kappa_c = 1/\sqrt{2}$ corresponds to $B(0) = 1$. The fact that there exists a monotonic relation between the GL parameter and the magnetic field at the vortex core center makes it possible to investigate

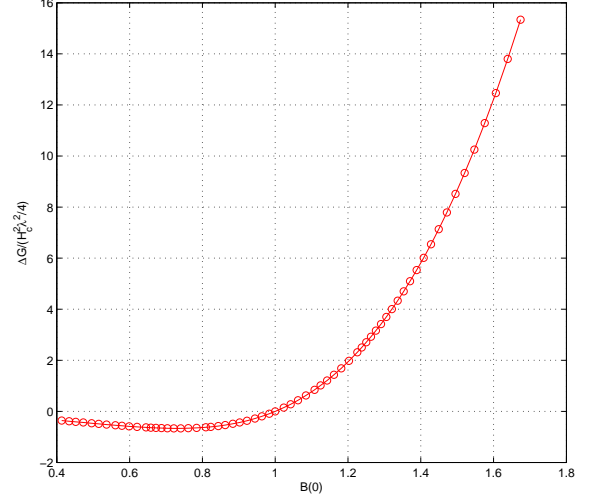


FIG. 2: (Color online) The Gibbs energy difference ΔG as function of the axial magnetic field $B(0)$. $\Delta G = 0$ represents the boundary between type-1/type-2 superconductors, and corresponds to $B(0) = 1$. $\Delta G \rightarrow 0^-$ as $B(0) \ll 1$ (see text).

the physical nature of vortex via $B(0)$ instead of the GL parameter κ , as we show below.

In order to study the stability of the vortex, we consider the Gibbs energy difference between the vortex state under the thermodynamic critical field and the Meissner state $\Delta G = G_{vortex}(H_c) - G_{Meissner}$. The same approach has been used to investigate the stability of vortex in a two-component superconductor¹⁷. With the definition of the Gibbs energy density $g = f - \mathbf{H} \cdot \mathbf{B}/4\pi$ and equation (1), ΔG can be written in the following form:

$$\begin{aligned} \Delta G &= \frac{H_c^2 \lambda^2}{4} \int_0^\infty \left[\frac{2}{\kappa^2} \left(\frac{d|\psi|}{d\rho} \right)^2 + \frac{1}{|\psi|^2} \left(-\frac{dB}{d\rho} \right)^2 - 2|\psi|^2 \right. \\ &\quad \left. + |\psi|^4 + (1-B)^2 \right] \rho d\rho \end{aligned} \quad (6)$$

Note that the Gibbs energy of the vortex state is decreasing with the increasing field. If $\Delta G > 0$, the material always behaves as type-1 superconductor since the Meissner state plays a dominate role. Once the magnetic flux penetrates into the superconductor in the form of vortex under certain value of the external field which is smaller than the thermodynamic critical field $H_e < H_c$, $\Delta G < 0$ is expected. $\Delta G = 0$ represents a boundary between the stable/unstable vortex state.

In figure 2, the variation of ΔG as a function of the magnetic field at the vortex center is presented. This is a remarkable result. It relates the local physical quantity, i.e., magnetic field at the center of a vortex, with the global thermodynamic stability of the vortex state. When $B(0) > 1$, vortex is thermodynamically unstable and $\Delta G > 0$, the superconductor is of type-1;

when $B(0) < 1$, vortex is thermodynamically stable and $\Delta G < 0$, the superconductor is of type-2. Zero of ΔG corresponds to $B(0) = 1$.

Asymptotic behavior of ΔG at $B(0) \ll 1$ can be estimated as follows. $B(0)$ is a monotonic decreasing function of the GL parameter κ and $B(0) \ll 1$ corresponds to $\kappa \gg 1$. In this case, contributions to ΔG from the vortex core ($\rho < 1/\kappa$) and far range ($\rho > 1$) can be ignored. In the region $1/\kappa \ll \rho \ll 1$, The approximate solution of the magnetic field is $B(\rho) \approx \sqrt{2}/\kappa \ln(1/\rho)$, $|\psi| \approx 1$. After some algebra, we find that, as $B(0) \ll 1$ ($\kappa \gg 1$), $\Delta G \propto -1/(\sqrt{2}\kappa) \rightarrow 0$.

We then derived the relation between the magnetic field at the vortex core center and the thermodynamic stability of the vortex. Due to the monotonic relationship between the core field $B(0)$ and the GL parameter κ , one can use $B(0)$ instead of κ to investigate the stability of the vortex. The critical case $B(0) = 1$ separate type-1 superconductor in which there exists no stable vortex ($B(0) > 1$) from type-2 material, in which there exist stable vortex ($B(0) < 1$).

Although the results of present work are based on the standard GL model calculations, we believe that, the conclusions have general meaning. The configurations of the vortices in a charged matter field always bear a certain resemblance to each other, including a normal vortex core in which the order parameter is suppressed to zero on the axis of the core, the restoration of the order parameter and the exclusion of magnetic field far from the vortex core. Taking into account the fact that the flux carried by a vortex is quantized, the difference between vortices originates from the difference of the magnetic field at the vortex core center. And it is natural that the physical nature of vortex is closely related to the quantity $B(0)$, the magnetic field at the center of a vortex.

The equivalence of our results and conventional methods which were used to classify superconductors can be demonstrated as follows. Physical interpretation of the zero of the surface energy is that the thermodynamic field is just the critical field at which vortex entry into the superconductor. The same case can be extended to the vortex geometry, where we set the external field equals to the thermodynamic critical field. Besides the emergence of a nontrivial topological charge due to the variation of the phase of the order parameter around the vortex core, when the external field equals to the field at the vortex center, $B(0) = 1$, $\Delta G = 0$ can be verified. Moreover, we found that, within the capabilities of our numerical simulation, no matter what the number of flux quanta is, field at the vortex core center $B(0) = 1$ and $\Delta G = 0$ at $\kappa = 1/\sqrt{2}$. This result implies that vortices do not interact in this regime. All these results proved qualitatively the equivalence of different types of schemes.

In summary, based on the thermodynamic argument, we have shown that the stability of vortex in a superconductor, or equally, the classification of the superconductors according to their magnetic response, has a close relation with the magnitude of the magnetic field at the vortex center. This finding provides a new criterion for the classification of the superconductors. Finally, we note that there exist models in high energy physics which are mathematically similar to the GL theory for superconductivity (e.g., the Abelian Higgs model¹⁸⁻²⁰). Our results can also be generalized to the investigation of the stability of topological vortex in these models.

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- that we have got the short-range asymptotics of the functions $|\psi|$ and B in Eq. (4). It is straightforward to obtain the derivatives of $|\psi|$ and B as functions of a , $B(0)$ from Eq. (4) near the core center. The problem can then be solved with collocation method and a , $B(0)$ are determined. In order to obtain precise solutions for a wider range of intrinsic parameter, we numerically solved the usual GL Eqs in which the vector potential \mathbf{A} and the order parameter Ψ are functions. Then the results are translated to the gauge invariant quantities $|\psi|$ and B .
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